TONBRIDGE SCHOOL

Scholarship Examination 2005

MATHEMATICS I

Tuesday 3rd May 2005
9.00 a.m.

Time allowed: 1 hour 30 minutes

Answer as many questions as you can.
Questions 1 to 5 are worth 8 marks each;
Questions 6 to 9 are worth 15 marks each.

All answers must be supported by adequate explanation.
Calculators may be used in any question.

1. A money-box contains only 2 pence and 5 pence coins. Altogether there are 36 coins with a total value of 129 pence. Use this information to make two simultaneous equations and solve them to find the number of 5 pence coins. [8 marks]

2. Consider the following sequence of fractions:

$$\frac{1}{1}, \frac{3}{2}, \frac{7}{5}, \frac{17}{12}, \ldots .$$

   a) What is the pattern relating a fraction in the sequence to its successor?

   b) Write down the next two fractions in the sequence.

   c) Work out the first six fractions in the sequence as decimals correct to three decimal places. What do you notice about your answers? [8]

3. In appropriate units, the maximum pressure, P, that a vertical pillar of height H can support before it collapses is given by the formula $P = \frac{C\pi^3}{H}$, where $\pi = 3.14159\ldots$ has its usual meaning and C depends on the mass of the pillar.

   a) If $C = 12.7$ and $H = 5.2$, find P.

   b) If $H = 17.3$ and $P = 9.3$, find C.

   c) In an experiment, the values of P, H, C are measured as $P = 62$, $H = 15$, $C = 30$. What value of π does the formula then give from these measurements? [8]

4. A *unit fraction* is one like $\frac{1}{4}$ with numerator 1.

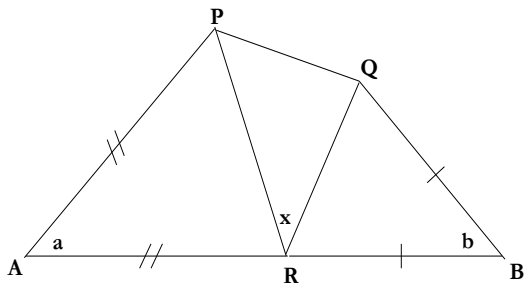
   a) Write 1 as the sum of three different unit fractions.

   b) By multiplying your answer in (a) by a suitable unit fraction, write $\frac{1}{6}$ as the sum of three different unit fractions.

   c) Use your answers to (a), (b) to write 1 as the sum of five different unit fractions.

   d) Write 1 as a sum of seven different unit fractions. [8]

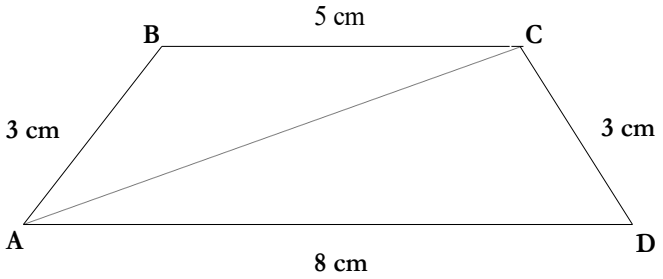
5. In the diagram below, APR and BRQ are isosceles triangles and ARB is a straight line.



   a) If $a = 40°$ and $b = 70°$, find x.

   b) In general, show that x is always the average (mean) of a and b. [8]

6. The diagram below shows an isosceles trapezium ABCD with sides of length 3 cm, 5 cm, 3 cm and 8 cm.



a) Use Pythagoras' theorem to show that the diagonal AC has a length which is a whole number of centimetres. (Hint: Drop a perpendicular from C to AD to create two right-angled triangles.)

b) Give an example of a rhombus with sides and both diagonals all having lengths which are whole numbers of centimetres. Explain your answer carefully. [15]

7. A famous formula of Einstein's predicts that if a 3 kg mass moves at a speed which is a fraction x of the speed of light, then its mass increases to a value y given by $y = \frac{3}{\sqrt{1-x^2}}$.

a) When $x = 0.2$, show that $y = 3.06$ kg.

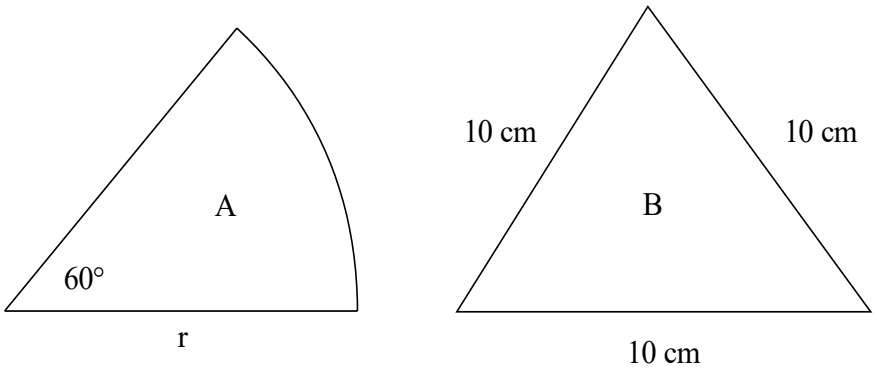
b) Calculate the values of y corresponding to $x = 0, 0.2, 0.4, 0.6, 0.8, 0.9, 0.95$.

c) Choosing sensible scales, plot a graph of y against x.

d) What value of x corresponds to $y = 6$?

e) What is it (i) about the shape of the graph and (ii) about the formula that suggests that the 3 kg mass cannot travel faster than the speed of light? [15]

8. The diagram below shows a circular sector (A) and an equilateral triangle (B). The sector has angle 60° and radius r and the equilateral triangle has sides of length 10 cm.



a) Find r if A and B have the same area.

b) Find r if A and B have the same perimeter. [15]

9. The terms F_1, F_2, F_3, . . . of the *Fibonacci sequence* are given by $F_1 = 1$, $F_2 = 1$, $F_3 = 2$, . . . where each term is the sum of the previous two.

a) Verify that $F_4 = 3$, $F_5 = 5$ and write down the next five terms of the Fibonacci sequence.

b) Consider the following statement about the Fibonacci sequence:

"If you square two successive terms of the sequence and add these squares together, then the answer is another term in the sequence".

Check this statement using your data in (a).
Given two successive terms as in the statement, explain carefully how you can predict which term of the sequence arises as the answer.

c) Use your answer to (b) to find x and y below:

i) $F_{50}^{\,2} + F_{51}^{\,2} = F_x$

ii) $F_y^{\,2} + F_{y+1}^{\,2} = F_{137}$.

[15]